Maximum-Likelihood spectral amplitude estimator [6]

Let $y[n] = x[n] + b[n]$ and $X(k) = A_k exp(\alpha_k)$ with the noise having a Gaussian distribution. The prior density function is (also see Ephraim and Malah suppression rule):

$$P(Y_k|a_k, \alpha_k) = \frac{1}{\pi\lambda_b(k)} exp\left\{-\frac{1}{\lambda_b(k)}|Y_k - a_k e^{j\alpha_k}|^2\right\}$$

The maximum-likelihood approach attempts to choose the parameter value that maximizes the parameterized pdf, that is the parameter value which is most likely to have caused the observation. The ML estimation is used for estimating an unknown parameter of a given pdf when no *a priori* information about it is available.

$$\begin{aligned}
\hat{A}_k &= \max_{a_k} \overline{P(Y_k|a_k, \alpha_k)} \\
&= \max_{a_k} P(Y_k|a_k) \\
&= \max_{a_k} \int_0^{2\pi} P(Y_k|a_k, \alpha_k) d\alpha_k \\
&= \frac{1}{2}\left\{|Y_k| + \sqrt{|Y_k|^2 - \lambda_b(k)}\right\} \\
&= \frac{1}{2}\left\{1 + \sqrt{\frac{|Y_k|^2 - \lambda_b(k)}{|Y_k|^2}}\right\}|Y_k| \\
&= \frac{1}{2}\left\{1 + \sqrt{\frac{\lambda_x(k)}{\lambda_x(k) + \lambda_b(k)}}\right\}|Y_k|
\end{aligned}$$

The performance of the algorithm during silent frames is not adequate because the starting assumption is that the signal is always present. The authors suggest a two-state soft-decision approach by using the binary hypothesis model:

$$\begin{array}{lll}
H_0: & \text{speech absent:} & Y_k = B_k \\
H_1: & \text{speech present:} & Y_k = A_k e^{j\alpha_k} + B_k
\end{array}$$

The MMSE solution is

$$\begin{aligned}\hat{A}_k &= \mathcal{E}[A_k|Y_k] \\ &= \mathcal{E}[A_k|Y_k,H_1]\times P(Y_k|H_1)+\mathcal{E}[A_k|Y_k,H_0]\times P(Y_k|H_0) \\ &= \mathcal{E}[A_k|Y_k,H_1]\times P(Y_k|H_1) \text{ since } \mathcal{E}[\text{speech mag}|\text{speech absent}]=0 \\ &\approx \tfrac{1}{2}\left\{|Y_k|+\sqrt{|Y_k|^2-\lambda_b(k)}\right\}\times P(Y_k|H_1)\end{aligned}$$

since $\mathcal{E}[A_k|Y_k,H_1]$ is the minimum variance estimate of A and the Maximum Likelihood estimate is asymptotically efficient.

$$P(Y_k|H_0)=\frac{2Y_k}{\lambda_b}exp(-\frac{Y_k^2}{\lambda_b}) \text{ (Rayleigh pdf for complex Gaussian noise)}$$

$$P(Y_k|H_1)=\frac{2Y_k}{\lambda_b}exp(-\frac{Y_k^2+A_k^2}{\lambda_w})I_0\left(\frac{2A_kY_k}{\lambda_b}\right) \text{ (Rician pdf for } A_ke^{j\alpha_k}+B_k)$$

Assuming $P(H_0)=P(H_1)=1/2$ and using Bayes' theorem,

$$P(Y_k|H_1)=\frac{exp(-\epsilon)I_0\left[2\sqrt{\epsilon\gamma}\right]}{1+exp(-\epsilon)I_0\left[2\sqrt{\epsilon\gamma}\right]}$$

denoting $\epsilon=A_k^2/\lambda_b$ to be the *a priori* SNR and $\gamma=Y_k^2/\lambda_b$ to be the *a posteriori* SNR.

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